ECE 330
POWER CIRCUITS AND ELECTROMECHANICS

LECTURES 4 AND 5
THREE-PHASE CONNECTIONS (1)

Acknowledgment-These handouts and lecture notes given in class are based on material from Prof. Peter Sauer's ECE 330 lecture notes. Some slides are taken from Ali Bazi's presentations

WHY THREE-PHASE?

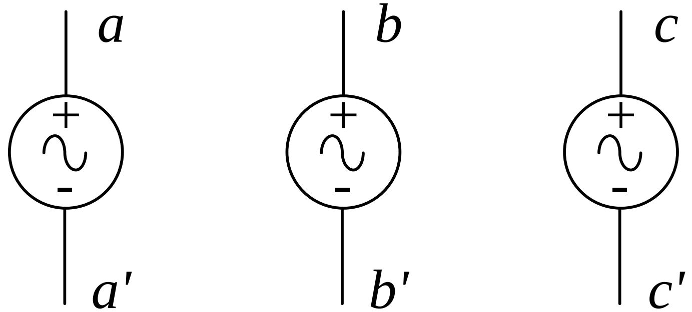
- Most effective in terms of equipment and wires.

- Produces smooth rotating magnetic field.

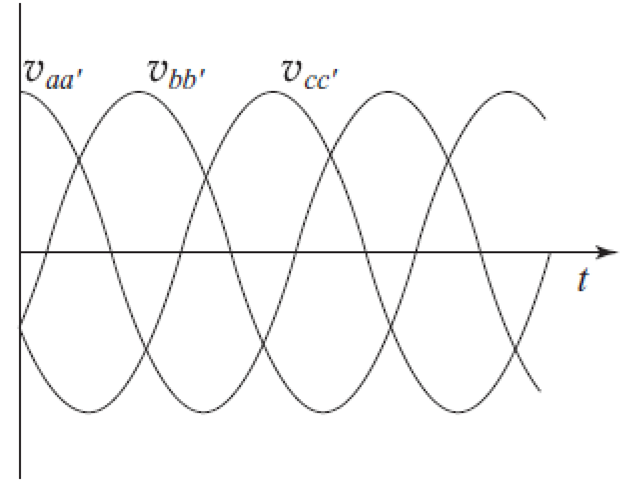
- Constant instantaneous power.

THREE-PHASE VOLTAGES

- The three phases are usually referred to as *abc*.
- Another standard notation is RYB (Red-Yellow-Blue).



- Voltages:
  - Same magnitude.
  - Same frequency.
  - Out of phase by 120°.



THREE-PHASE VOLTAGES

- Positive-sequence voltages are most common (*abc*).

$$v_{aa'}(t) = V_m \cos(\omega t)$$

$$v_{bb'}(t) = V_m \cos(\omega t - 120^o)$$

$$v_{cc'}(t) = V_m \cos(\omega t + 120^o)$$

$$\bar{V}_{aa'} = \frac{V_m}{\sqrt{2}} \angle 0^o$$

$$\bar{V}_{bb'} = \frac{V_m}{\sqrt{2}} \angle -120^o$$

$$\bar{V}_{cc'} = \frac{V_m}{\sqrt{2}} \angle +120^o$$

- Negative-sequence voltages, flip *b* and *c* (*acb*).

$$v_{aa'}(t) = V_m \cos(\omega t)$$

$$v_{bb'}(t) = V_m \cos(\omega t + 120^o)$$

$$v_{cc'}(t) = V_m \cos(\omega t - 120^o)$$

$$\bar{V}_{aa'} = \frac{V_m}{\sqrt{2}} \angle 0^o$$

$$\bar{V}_{bb'} = \frac{V_m}{\sqrt{2}} \angle +120^o$$

$$\bar{V}_{cc'} = \frac{V_m}{\sqrt{2}} \angle -120^o$$

THREE-PHASE VOLTAGES

- If phase *a* has angle θ_v, the positive sequence is (you practice the negative sequence):

$$v_{aa'}(t) = V_m \cos(\omega t + \theta_v)$$

$$v_{bb'}(t) = V_m \cos(\omega t + \theta_v - 120^o)$$

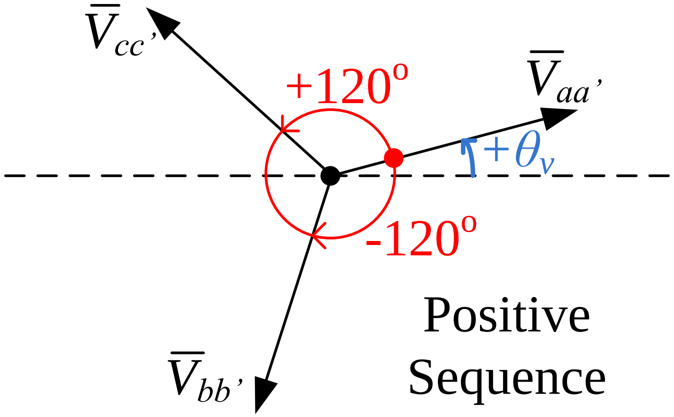
$$v_{cc'}(t) = V_m \cos(\omega t + \theta_v + 120^o)$$

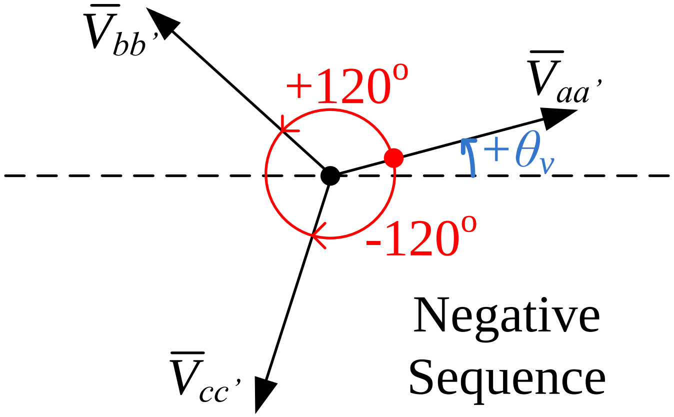
$$\bar{V}_{aa'} = \frac{V_m}{\sqrt{2}} \angle \theta_v$$

$$\bar{V}_{bb'} = \frac{V_m}{\sqrt{2}} \angle \left(\theta_v - 120^o\right)$$

$$\bar{V}_{cc'} = \frac{V_m}{\sqrt{2}} \angle \left(\theta_v + 120^o\right)$$

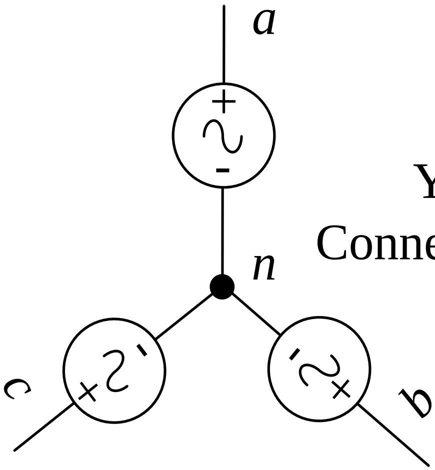
- Phasor representation:



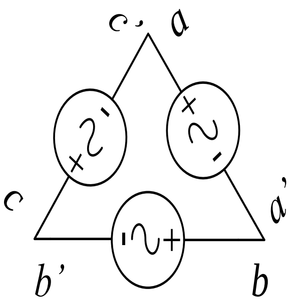


THREE-PHASE CONNECTIONS

- In general, positive sequence is used unless otherwise specified.
- Two main connections:
  - Wye (Y)
  - Delta (Δ)



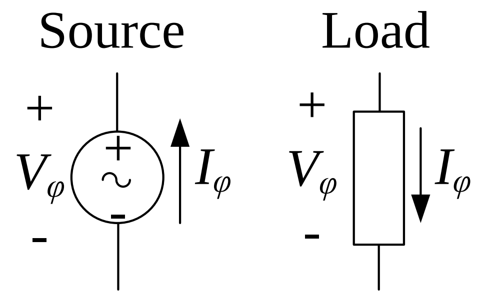
Y Connection



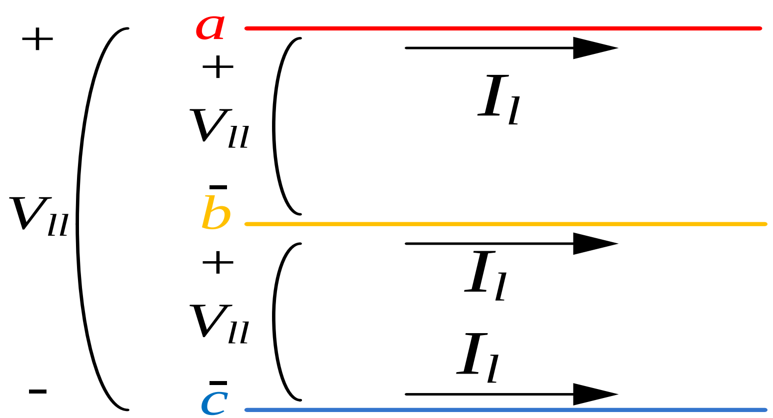
Delta Connection

PHASE AND LINE

- Phase voltage ($V_{ø}$) and phase current ($I_{ø}$) are defined across an element:



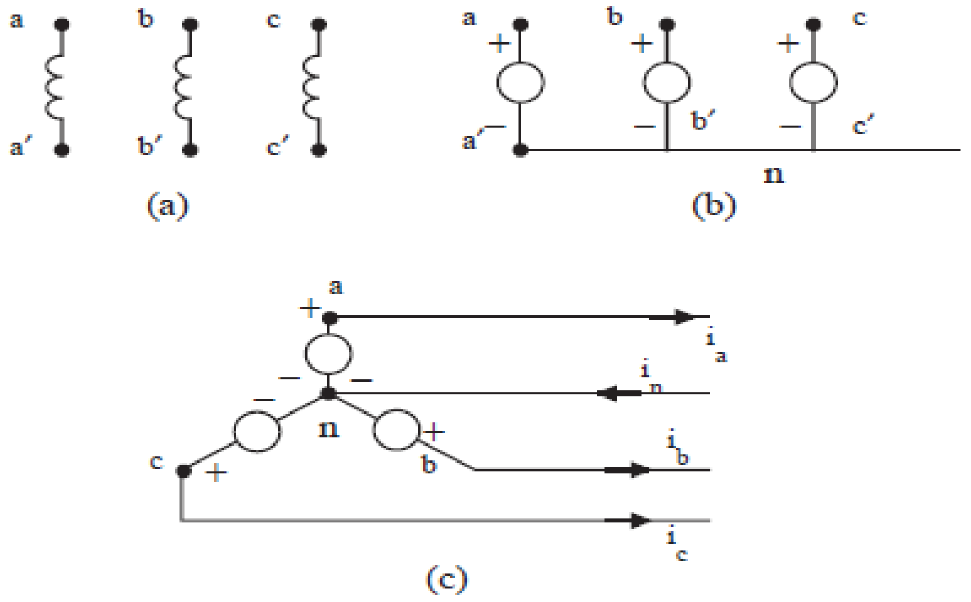
- Line-to-line (or line) voltage (V_{ll}) and line current (I_l) are defined on the line:



THREE-PHASE CONNECTION

WYE CONNECTION:

The three windings are represented by coils. In the wye connection, terminals a', b', and c' are joined and labeled as terminal n, also called the *neutral terminal*

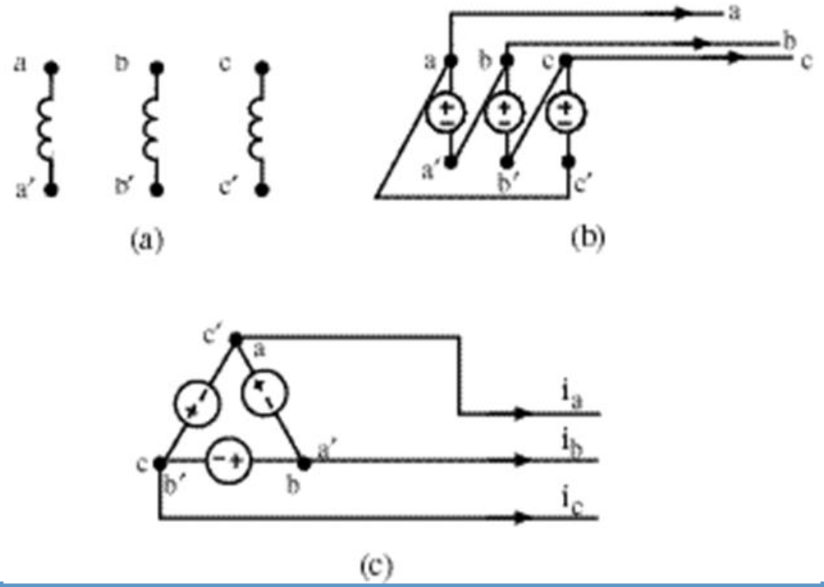


THREE-PHASE CONNECTION

DELTA CONNECTION:

In the delta connection, terminal a' is connected to b, and b' to c Finally, c' can be connected to terminal a .

The voltage across $v_{ac'} = v_{aa'}(t) + v_{bb'}(t) + v_{cc'}(t)$ and is identically equal to zero



Y-CONNECTION

- Voltages can be line-to-line (V_{ll}) or line-to-neutral (V_{ln}).
- Currents can be line (I_l) or phase ($I_{ø}$).
- Line-to-line voltages can be referred to as line voltages (V_l).
- Line-to-neutral voltages can be referred to as phase voltages ($V_{ø}$).

Y-CONNECTION

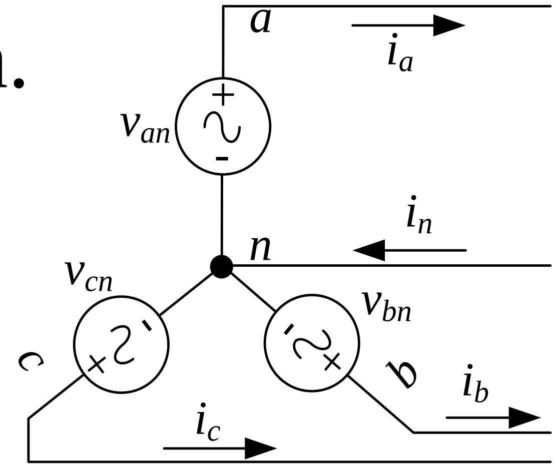
- Line-to-line voltages are v_{ab}, v_{bc}, and v_{ca}.

- Line-to-neutral voltages are v_{an}, v_{bn}, and v_{cn} which are also $\bar{V}_{an} = V_{\phi}\angle 0^o, \bar{V}_{bn} = V_{\phi}\angle -120^o, \bar{V}_{cn} = V_{\phi}\angle +120^o.$

- Example: $\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn}$ and so on.

$$\bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn}$$

$$\bar{V}_{ca} = \bar{V}_{cn} - \bar{V}_{an}$$

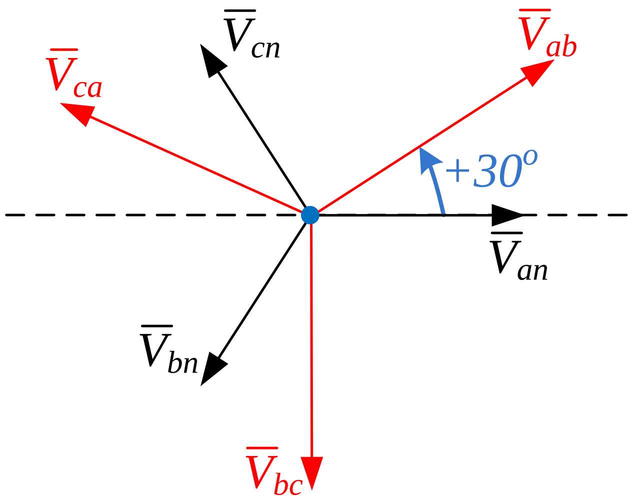


Y-CONNECTION

- Thus:

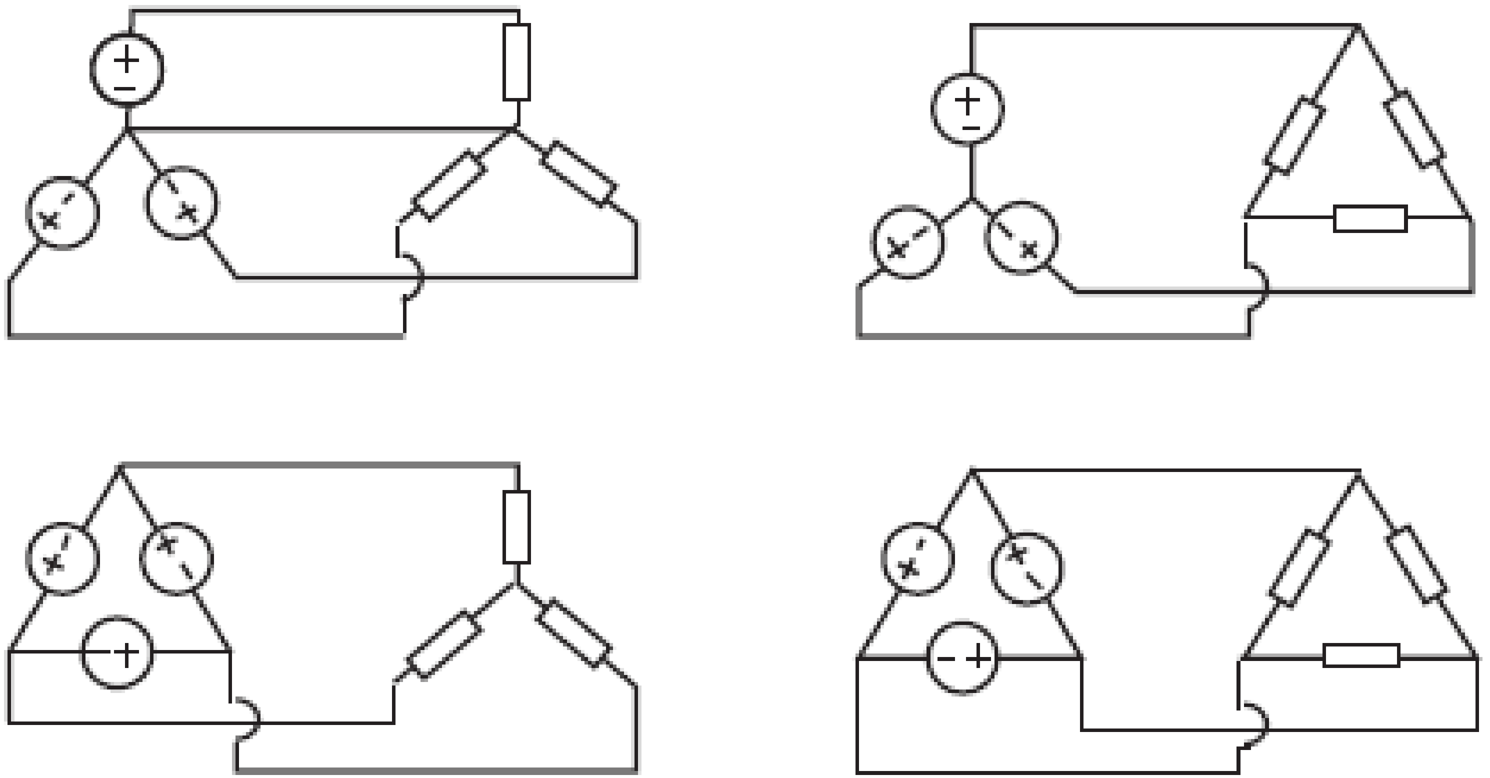
$$\begin{cases} \overline{V}_{ab} = V_{\phi}\angle 0^{o} - V_{\phi}\angle -120^{o} \\ \overline{V}_{bc} = V_{\phi}\angle -120^{o} - V_{\phi}\angle +120^{o} \\ \overline{V}_{ca} = V_{\phi}\angle +120^{o} - V_{\phi}\angle 0^{o} \end{cases}$$

$$\begin{cases} \overline{V}_{ab} = \sqrt{3}V_{\phi}\angle 30^{o} \\ \overline{V}_{bc} = \sqrt{3}V_{\phi}\angle -90^{o} \\ \overline{V}_{ca} = \sqrt{3}V_{\phi}\angle +150^{o} \end{cases}$$



$$\begin{cases} |V_{ll}| = \sqrt{3}\,|V_{\phi}| \\ \overline{I}_{l} = \overline{I}_{\phi} \end{cases}$$

Y OR Δ CONNECTION?



Y OR Δ CONNECTION?

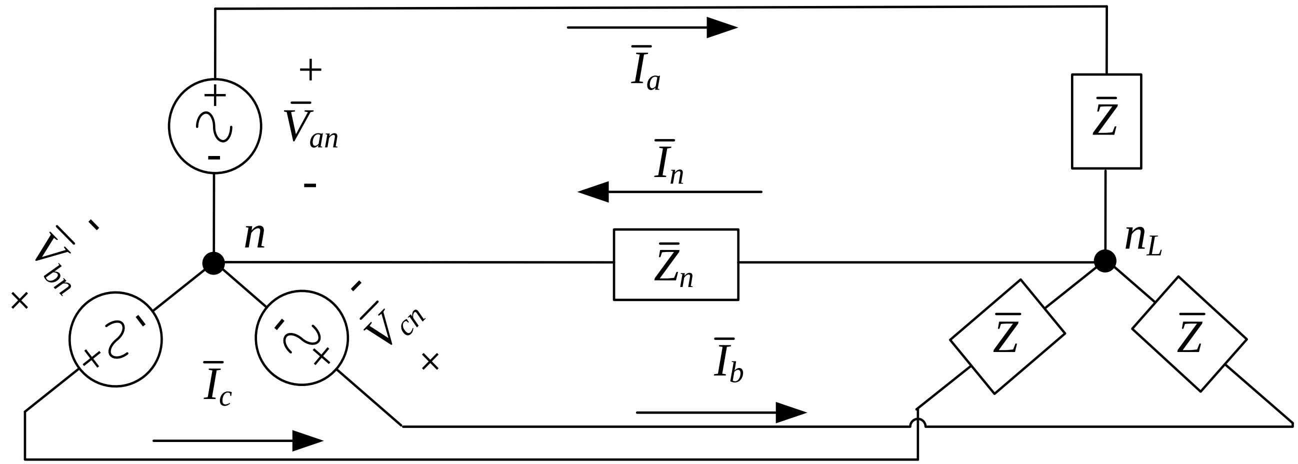
In distribution, the secondary winding is often in wye, as it provides a neutral for single or three phase loads.

The primary is often in delta, as it allows 3rd harmonics to flow within the transformer and prevent it flowing into the supply.

Y OR Δ CONNECTION?

- In Y connection, voltage across each coil is smaller than the line voltage, so it is preferable where high line voltage are used as in high voltage power systems, to protect coils from high voltage.
- In delta connection current passing through coils are smaller than line currents, therefore coils can be rated for smaller current.

Y-SOURCE, Y-LOAD



$$\bar{V}_{an} = \bar{I}_a \bar{Z} + \bar{I}_n \bar{Z}_n$$

$$\bar{V}_{bn} = \bar{I}_b \bar{Z} + \bar{I}_n \bar{Z}_n$$

$$\bar{V}_{cn} = \bar{I}_c \bar{Z} + \bar{I}_n \bar{Z}_n$$

$$add : 0 = \left(\bar{I}_a + \bar{I}_b + \bar{I}_c\right)\bar{Z} + 3\bar{I}_n \bar{Z}_n$$

$$but\ by\ KCL, \bar{I}_a + \bar{I}_b + \bar{I}_c = \bar{I}_n$$

$$\Rightarrow \bar{I}_n \left(\bar{Z} + 3\bar{Z}_n\right) = 0$$

$$\Rightarrow \bar{I}_n = 0$$

$$\Rightarrow \bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}}, \bar{I}_b = \frac{\bar{V}_{bn}}{\bar{Z}}, \bar{I}_c = \frac{\bar{V}_{cn}}{\bar{Z}}$$

Y-SOURCE, Y-LOAD

- For a load impedance $\bar{Z} = Z\angle\theta,$

$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}} = \frac{V_\phi\angle 0}{Z\angle\theta} = I_\phi\angle -\theta = I_l\angle -\theta$$

$$\bar{I}_b = \frac{\bar{V}_{bn}}{\bar{Z}} = \frac{V_\phi\angle -120^o}{Z\angle\theta} = I_\phi\angle\left(-120^o - \theta\right) = I_l\angle\left(-120^o - \theta\right)$$

$$\bar{I}_c = \frac{\bar{V}_{cn}}{\bar{Z}} = \frac{V_\phi\angle +120^o}{Z\angle\theta} = I_\phi\angle\left(+120^o - \theta\right) = I_l\angle\left(+120^o - \theta\right)$$

Y-SOURCE, Y-LOAD

$$\bar{S}_{3\phi} = \bar{S}_a + \bar{S}_b + \bar{S}_c + \bar{S}_n,\ if\ \ \bar{S}_n = 0\ i.e.\ balanced\ \ load$$

$$= \bar{V}_{an}\bar{I}_a^* + \bar{V}_{bn}\bar{I}_b^* + \bar{V}_{cn}\bar{I}_c^*$$

$$= \bar{V}_{an}\bar{I}_a^* + \bar{V}_{bn}\bar{I}_b^* + \bar{V}_{cn}(-\bar{I}_a^* - \bar{I}_b^*)$$

$$= \left(\bar{V}_{an} - \bar{V}_{cn}\right)\bar{I}_a^* + \left(\bar{V}_{bn} - \bar{V}_{cn}\right)\bar{I}_b^*$$

$$= \bar{V}_{bc}\bar{I}_b^* - \bar{V}_{ca}\bar{I}_a^*$$

Y-SOURCE, Y-LOAD

$$\bar{S}_{3\phi} = \sqrt{3}V_{\phi}\angle -90^{o}\, I_{\phi}\angle\left(+120^{o} + \theta\right) - \sqrt{3}V_{\phi}\angle 150^{o}\, I_{\phi}\angle\left(\theta\right)$$

$$= \sqrt{3}V_{\phi}I_{\phi}\left[1\angle\left(+30^{o} + \theta\right) - 1\angle\left(+150^{o} + \theta\right)\right]$$

$$= \sqrt{3}V_{\phi}I_{\phi}\left[\sqrt{3}\cos\left(\theta\right) + j\sqrt{3}\sin\left(\theta\right)\right]$$

$$\Rightarrow \bar{S}_{3\phi} = 3V_{\phi}I_{\phi}\angle\theta$$

$$\Leftrightarrow \bar{S}_{3\phi} = \sqrt{3}V_{ll}I_{l}\angle\theta$$

Y-SOURCE, Y-LOAD

- Therefore,

$$\bar{S}_{3\varphi} = \sqrt{3} V_{ll} I_l \angle \theta$$

$$\Rightarrow P_{3\varphi} = \sqrt{3} V_{ll} I_l \cos(\theta)$$

$$\Rightarrow Q_{3\varphi} = \sqrt{3} V_{ll} I_l \sin(\theta)$$

- Note that (V_{ll}) can also be written as (V_l).

READING MATERIAL

- Reading material: Sections 2.5.1 and 2.5.2.

- Recommended reading for next time: Continue section 2.5.2 and section 2.5.3.